(13)

ARL-AERO-TM-372

AR-004-021

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

AERONAUTICAL RESEARCH LABORATORIES

MELBOURNE, VICTORIA

Aerodynamic Technical Memorandum 372

TWO-DIMENSIONAL HELMHOLTZ FLOW
ABOUT A FLAT PLATE AND RELATED SHAPES

by

TON TRAN-CONG

and

C.A. MARTIN

Approved for Public Release

DTIC
SELECTE
OCT 3 1 1985
E

MAY 1985

AD-A160 806

DTIC FILE COPY

AR-004-021

DEPARTMENT OF DEFENCE
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION
AERONAUTICAL RESEARCH LABORATORIES

Aerodynamic Technical Memorandum 372

TWO-DIMENSIONAL HELMHOLTZ FLOW

ABOUT A FLAT PLATE AND RELATED SHAPES

by

TON TRAN-CONG

and

C.A. MARTIN

SUMMARY

      Various configurations of two-dimensional Helmholtz flow about a flat plate are obtained with a free streamline theory using the flow inside a semi-circular disc;   The flow can vary continuously from fully-attached Joukowski flow to the commonly-known fully-separated Helmholtz flow. Some general results for an arbitrary aerofoil are also obtained. Variations of the technique give the flow about flat plates with flaps and other related configurations.

CONTENTS

1. INTRODUCTION

The incompressible inviscid flow about a flat plate is basic to the theory of aerofoils in aerodynamics. Fully-attached flow around a flat plate is obtained with conformal mapping from flow around a circular cylinder with the application of the Joukowski condition to fix the trailing streamline on the trailing edge of the flat plate. Realizing that it is also possible to obtain free-stream flow at the rear of the plate, Schmieden [1] proposed a method to generate this kind of flow. However Schmieden's method can only generate free-stream flows which are not allowed to reunite again (except at infinity). Therefore it is not possible to examine the evolution of the flow from the fully-attached state to the fully-separated state using the above two theories.

In this Memo an alternative method for generating the flow about a flat plate is presented. The method is more general than the above two in the sense that it yields the two flows as limiting cases of this more general flow. The present method permits a continuous variation from the fully-attached (called here Joukowski) flow through to the separated (called here the Helmholtz-Schmieden) flow to the commonly known Helmholtz flow of figure 4, as given in standard text such as [2]. The method uses a semi-circular σ-plane with a doublet-vortex at an interior point D chosen such that different conditions of separation and closure of the free stream-lines are obtained.

Although the separated flow (with or without reunion) about an arbitrary aerofoil cannot yet be constructed, the existence of these flows can be proved, hence substantiating a number of conjectures made by Schmieden.

By altering the complex velocity function we can also generate a number of variant flows among which are flows about a flat plate with flaps and flow with a leading edge separation bubble.

The flows described by Hurley and Ruglen [3], which have beenconsidered more recently by Saffman and Tanveer [4], are also included. It is also noted that separated flow with reunion around a circular cylinder has been obtained numerically by Southwell and Vaisey [5], using relaxation method.

2. FLOW ABOUT A FLAT PLATE

The fully attached incompressible, inviscid flow about
a flat plate is familiar and is obtained by a conformal transformation
of flow about a circular cylinder. Fully-separated flows have been
worked out by Schmieden [1].   However, due to Schmieden's choice of
the transformed τ-plane, the separated flow situation is completely cut
off from the fully-attached flow situation. Here the problem is
reformulated such that a continuous variation from one case to the other
is possible: In this way the two extreme cases become the limits of a
more general flow regime.

Following Schmieden, we define a complex velocity
potential w by

$$w = \phi + i\psi \qquad (\phi, \ \psi \ \text{real}), \tag{1}$$

and a complex variable ζ is introduced

$$\zeta = \ln \frac{dz}{dw} = \ln \frac{1}{q} + i\theta, \tag{2}$$

where q and θ are the magnitude and angle of the velocity vector in the
z plane. The corresponding flows are shown in Figs 1 and 2.

Introducing a σ plane (similar to the τ plane in
Schmieden's paper, but with less restriction on its use) such that

$$\frac{dz}{dw} = e^{\zeta} = C \ \frac{(\sigma - \bar{\sigma}_A) \ (\sigma - \sigma_B)}{(\sigma - \sigma_A) \ (\sigma - \bar{\sigma}_B)} , \tag{3}$$

The flow shown in Figure 3 is obtained. The complex
scaling constant C determines the size and orientation of the flow in
the z-plane. In the figure, the upper semi-circular disc of the σ plane
is the image of the notched strip of the ζ plane. The upper semi-circle
corresponds to the constant direction paths in the z plane and the
segment (-1, +1) of the real σ axis corresponds to the free stream-lines
CD and ED of the z plane. The line of $\psi = 0$ in the z plane is the
free stream-line(s) and corresponds to the line(s) $\psi = 0$ in the σ plane
which is DABCWD and DAED.

Point W, the inflection point of the free streamline DABCWD is chosen to be at the center of the circle of the σ plane. In this way σ_A is equal to $(-\bar{\sigma}_B)$.

By putting D on the real segment CE of the σ plane the kind of free-stream flow given previously by Schmieden is obtained although the mathematics is less complex.

The flow from D is produced by a doublet plus a quadruplet such that $w(\sigma)$ is real on the boundary of the semi-circular disc and the streamline D to A divides at A and then follows the boundary to return to D as shown in Fig. 3. The complex potential of this flow is given by

$$w(\sigma) = \left\{ \frac{1}{(\sigma-\sigma_D)^2} + \frac{\sigma^2}{(1-\bar{\sigma}_D\sigma)^2} - \frac{4\mu}{(\sigma-\sigma_D)} - \frac{4\mu\sigma}{(1-\bar{\sigma}_D\sigma)} \right\}, \quad (4a)$$

the constant μ is determined such that σ_A is a stagnation point.

The flow in the z plane is obtained by integrating dz, as

$$z = \int dz = \int \frac{dz}{dw} \frac{dw}{d\sigma} d\sigma. \quad (5)$$

The results are as already given in Schmieden's paper: When D is very close to E on the σ plane the resulting flow is as shown in Figure 4. For D located between E and W the flow is as shown in Figure 1. For D very close to W the flow closes at infinity as shown in Figure 5. σ_D cannot be negative or else the two free streamlines BCWD and ED intersect each other presenting a physically impossible overlap of the flow field.

An extension of Schmiedens approach is now made in which the point D is moved up from the real axis of the σ plane. The derivative dz/dw is still given by formula (3) but the complex potential $w(\sigma)$ is now given by

(4)

$$w(\sigma) = \left\{ e^{i\beta} \left(\frac{1}{\sigma - \sigma_D}\right) + e^{i\beta} \left(\frac{\sigma}{1-\sigma_D\sigma}\right) \right.$$

$$+ e^{-i\beta} \left(\frac{1}{\sigma - \bar{\sigma}_D}\right) + e^{-i\beta} \left(\frac{\sigma}{1-\bar{\sigma}_D\sigma}\right)$$

$$\left. + i\gamma \ \ln \frac{(\sigma - \sigma_D)(1 - \sigma_D\sigma)}{(\sigma - \bar{\sigma}_D)(1 - \bar{\sigma}_D\sigma)} \right\} \qquad (4b)$$

where β, γ are real numbers which represent a doublet plus a vortex at σ_D and its three image points. In this way the multiple valued function $w(\sigma)$ is still real on the boundary of the semi-circular disc (the plane is cut along the streamline DW to make $w(\sigma)$ single valued). The real numbers β and γ must make A a stagnation point in the σ plane and also make the point F in the σ plane correspond to one single point z_F in the z plane. This point z_F is the point at which closure of the free stream-lines occurs. These two conditions require respectively

$$\gamma = \frac{\mathrm{Im} \ \left(e^{i\beta} \left[\dfrac{\sigma_A}{(\sigma_A-\sigma_D)^2} - \dfrac{\bar{\sigma}_A}{(\bar{\sigma}_A-\sigma_D)^2} \right] \right)}{\mathrm{Re} \ \left(\dfrac{\sigma_A}{\sigma_A-\sigma_D} - \dfrac{\bar{\sigma}_A}{\bar{\sigma}_A-\sigma_D} \right)} \qquad (6)$$

and

$$\frac{i\gamma}{e^{i\beta}} = \left\{ \frac{d}{d\sigma} \left[\ln \frac{dz}{dw} \right] \right\}_{\sigma=\sigma_D} \qquad (7)$$

Unfortunately the last two equations (6) and (7) cannot both be satisfied with any arbitrarily chosen value of σ_D inside the semi-circular disc. To satisfy both equations (6) and (7) the point σ_D must follow a curve joining $\sigma = 0$ and $\sigma = i$ (which is different for each angle of attack), as in Figure 6.

It is seen that for each given σ_A and $\sigma_B = -\bar{\sigma}_A$, the difference $h(\sigma_D)$ between the two values of γ given by equations (6) and (7),

$$h(\sigma_D) = \frac{Im\left(e^{i\beta}\left[\dfrac{\sigma_A}{(\sigma_A-\sigma_D)^2} - \dfrac{\bar{\sigma}_A}{(\bar{\sigma}_A-\sigma_D)^2}\right]\right)}{Re\left(\dfrac{\sigma_A}{\sigma_A-\sigma_D} - \dfrac{\bar{\sigma}_A}{\bar{\sigma}_A-\sigma_D}\right)} + 2ie^{i\beta}\sigma_D\left[\dfrac{1}{\sigma_D^2-\bar{\sigma}_A^2} - \dfrac{1}{\sigma_D^2-\sigma_A^2}\right]$$

tends to $+\infty$ and $-\infty$ when the argument of σ_D tends to 0 and to π respectively while the magnitude $0<|\sigma_D|<1$ is kept constant. This is so as the last term remains finite and the first term is approximated by $-\cos\beta / Im(\sigma_D)$.

Since $h(\sigma_D)$ is continuous inside the semi-circular disc there must be a continuous line joining $\sigma = 0$ and σ_A such that $h(\sigma_D)$ is zero when σ_D lies on such a line. As the angle of attack is a continuous function of σ_D and σ_A there exists a continuous line joining $\sigma = 0$, $\sigma = i$ for each angle of attack such that the function $h(\sigma_D)$ is zero on this line,

Therefore with each angle of attack the free-stream flow can vary from fully open as in Figure 4 through Figure 1 and closes as in Figure 5. Afterwards the flow closes at a finite point z_F as in Figure 7. As σ_D and σ_A tend to i, z_F tends to z_E, hence the limit is a fully attached Joukowski flow as shown in Figure 8. The vortex strength for the closed flow increases as the Joukowski flow is approached. Hence the closed flow may be regarded as a "reduced circulation" flow.

Note that the proof of the existence of the locus for σ_D such that $h(\sigma_D) = 0$ holds irrespective of the form of the function dz/dw, as long as we have

$$\lim_{\sigma_D \to (-r)} (\beta) \quad . \quad \lim_{\sigma_D \to r} (\beta) \quad < 0 \quad ,$$

with $\beta(\sigma_D)$ determined from only equation (7).

The inflection point W of the free streamlines moves from the upper streamline (between B and D) down to the lower streamline (between D and E) as the flow closes. In the full Helmholtz flow we have B,C,W becoming one point and in the full Joukowski flow we have E,W,F,C, becoming one single point.

At any incidence of the flat plate there is no drag from the Joukowski flow until the two free stream-lines separate at infinity. Thereafter a growing zone with static pressure appears on the rear of the plate giving rise to drag increasing to the full value of Helmholtz flow.

3.   FLOW ABOUT AN ARBITRARY AEROFOIL

In this section the proof is given of the existence of separated flows about an arbitrary aerofoil. This result was conjectured by Schmieden [1] by physical reasoning.

For such a flow about an arbitrary aerofoil profile as shown in Figure 9, the flow in the ζ plane can be constructed as in Figure 10. A flow in the χ plane is then given by

$$e^\xi = C \left(\frac{\chi^2 - \chi^2_B}{\chi^2 - \chi^2_A} \right) ,$$

with C being a complex scaling constant. Obviously the aerofoil boundary in the χ plane is not a semi-circle of unit radius but is distorted as depicted in Figure 11.

Using Riemann's mapping theorem, the area bounded by the curve ACWDEA can be mapped onto the upper semi-circular section in the σ plane of Figure 3 such that $\sigma_W = 0, |\sigma_A| = |\sigma_C| = |\sigma_E| = 1$ and $d\chi / d\sigma > 0$ at $\chi = 0$. In this way the transformation $\chi(\sigma)$ is given by

$$\chi(\sigma) = \sum_{n=1}^{\infty} b_n \sigma^n \tag{8}$$

where b_i are the real-valued coefficients of the series and $b_1 > 0$. The series so defined is convergent for all $|\sigma| \leq 1$.

The result presented above covers Schmieden's conjecture: The separated flow around any given aerofoil corresponds to a separated flow around a flat plate. The correspondence is given in equation (8).

The composite function $\xi(\chi(\sigma))$ for this flow is thus defined for all σ inside the upper semi-circular disc. By writing

$$\xi(\chi(\sigma)) \approx \ln \left(\frac{\sigma - \bar{\sigma}_A}{\sigma - \sigma_A}\right) + i \quad \phi(\chi(\sigma))$$

it can be proved that the function $\phi(\chi(\sigma))$, which is

$$i \phi(\chi(\sigma)) = \ln \left(\frac{\chi - \bar{\chi}_A}{\chi - \chi_A}\right) - \ln \left(\frac{\sigma - \bar{\sigma}_A}{\sigma - \sigma_A}\right) + \ln \left(\frac{\chi - \chi_B}{\chi - \bar{\chi}_B}\right) + \text{constant,}$$

is analytic on the semi-circular disc and that $\phi(\chi(\sigma))$ is real for $-1 \leq \sigma \leq 1$. Therefore $\xi(\chi(\sigma))$ takes the form

$$\xi(\sigma) = \ln \left(\frac{\sigma - \bar{\sigma}_A}{\sigma - \sigma_A}\right) + i \sum_{n=1}^{\infty} n a_n \sigma^n + i a_0 , \tag{9}$$

all a_1, a_2, a_3, ... real, as previously given by Schmieden. The *constant* a_0 is real for real σ_D and is complex otherwise.

The condition for the stagnation point to be at A remains the same as for a flat plate and is given by equation (6).

The closure condition (7) now becomes

$$\frac{i\gamma}{e^{i\beta}} = \left\{\frac{d\xi}{d\sigma}\right\}_{\sigma = \sigma_D} ,$$

or

$$\frac{i\gamma}{e^{i\beta}} = \left\{\frac{1}{\sigma_D - \bar{\sigma}_A} - \frac{1}{\sigma_D - \sigma_A} + i \sum_{n=1}^{\infty} n^2 a_n \sigma_D^{n-1}\right\} . \tag{10}$$

When σ_D sweeps a semi-circular arc described by $\sigma_D = re^{i\theta}$ ($0 < r < 1$, r kept constant, θ increasing from 0 to π) the first two terms being $-2i$ Im $(\sigma_A) / [(\sigma_D - \sigma_A)(\sigma_D - \bar{\sigma}_A)]$ do not contribute to the increase in β but the third term may alter the value of β.

The right hand side of equation (10) is the product of $d\xi/d\chi$ and $d\chi/d\sigma$. Since $\chi(\sigma)$ is a one-to-one and onto conformal mapping the derivative $d\chi/d\sigma$ cannot vanish anywhere on the disc $|\sigma| < 1$. On the other hand, the derivative

$$\frac{d\xi}{d\chi} = 2\chi \left(\frac{1}{\chi^2 - \chi_B^2} - \frac{1}{\chi^2 - \chi_A^2} \right) = 2\chi \frac{\chi_B^2 - \chi_A^2}{(\chi^2 - \chi_B^2)(\chi^2 - \chi_A^2)}$$

which changes sign once for χ travelling on the real segment $(-1,1)$.

Therefore β changes its value by π when σ_D sweeps the semi-circular arc described by $\sigma_D = re^{i\theta}$ ($0 < r < 1$, r kept constant, θ increasing from 0 to π).

Thus a proof similar to that of the last section can be constructed to prove that there exist a continuous line for σ_D such that the pair of equations (6) and (7) are satisfied on that line.

Hence with any arbitrary aerofoil at a given angle of attack, the flow can vary continuously from full Joukowski flow to full Helmholtz flow.

It is worthwhile noting that the curvature k for a free stream-line is given by

$$k = \frac{d\theta}{|dz|} = -i \left\{ \frac{d\xi}{d\sigma} \quad \frac{d\sigma}{dw} \right\}_{\text{for real } \sigma} \tag{11}$$

hence

$$k = \left\{ -2\text{Im} \left(\frac{1}{\sigma - \sigma_A} \right) + \sum_{n=1}^{\infty} n^2 a_n \sigma^{n-1} \right\} \frac{d\chi}{d\sigma} \quad \frac{d\sigma}{dw} .$$

As σ_D tends to 0, σ_A tends to $(ie^{-\alpha/2})$ and k tends to zero. Therefore the quantity inside the accolades must vanish. Hence

$$a_1 = 2 \cos \left(\frac{\alpha}{2}\right) \tag{12}$$

for the flows which close at infinity, as previously observed by Schmieden.

For a separated flow with reunion of the two free stream-lines the circulation around the wing is $2\pi\gamma$, where γ is given by the system of equations (6) and (7). The free-stream velocity (at the point D) is given by

$$u_\infty - iv_\infty = \left(\frac{dw}{dz}\right)_D = \frac{1}{c} \left(\frac{X_D^2 - X_A^2}{X_D^2 - X_B^2}\right)$$

and the pure lift (there is no drag here) is given by

$$\rho 2\pi\gamma \left| \frac{1}{c} \frac{(X_D^2 - X_A^2)}{X_D^2 \, X_B^2} \right| ,$$

where ρ is the density of the fluid. As the two stream-lines open, this pure lift changes smoothly into lift and drag caused by a stagnation zone behind the aerofoil.

The question of generating a free-stream flow about an arbitrarily given aerofoil still has not been satisfactorily answered. Towards this aim it can be noted that if $\xi(\sigma)$ is given its most general form

$$\xi(\sigma) = \ln \left(\frac{\sigma - \bar{\sigma}_A}{\sigma - \sigma_A}\right) + \int_0^1 \ln \left(\frac{\sigma - e^{it}}{\sigma - e^{-it}}\right) \, df(t) + complex \; constant$$

where f(t) is a function with bounded variation defined for all real t with $o \leq t \leq 1$, then each flow regime corresponds to a function f(t). However the construction of such a function f(t) from a given aerofoil shape and σ_D is very complicated. Possible techniques for this construction are currently under investigation.

From the above considerations, it would appear that the σ-plane technique would also be appropriate for the solution of two-dimensional free stream-line flow about an arbitrary single body.

4. VARIANTS OF THE FLOW

By making some minor changes to dz/dw different flows are speculated from the same basic σ-plane.

a) Flow about a flat plate with leading edge flap: This is obtained with

$$\frac{dz}{dw} = C \left(\frac{\sigma-\tilde{\sigma}_A}{\sigma-\sigma_A}\right) \left(\frac{\sigma-\sigma_B}{\sigma-\tilde{\sigma}_B}\right)^m ,$$

where the real number mπ is the flap deflection angle in radians. The flow is illustrated in Figures 12a and 12b. The two trailing stream lines can also close if D is off the real axis of the σ plane.

b) Flow about a flat plate with trailing edge flap. This is obtained with

$$\frac{dz}{dw} = C \left(\frac{\sigma-\tilde{\sigma}_A}{\sigma-\sigma_A}\right) \left(\frac{\sigma-\sigma_G}{\sigma-\tilde{\sigma}_G}\right)^n ,$$

where the real number nπ denotes the flap deflection angle in radians. The flow is given in Figure 13a and 13b.

c) Flow about a flat plate with leading and trailing edge flaps: This is obtained with

$$\frac{dz}{dw} = C \left(\frac{\sigma-\tilde{\sigma}_A}{\sigma-\sigma_A}\right) \left(\frac{\sigma-\sigma_B}{\sigma-\tilde{\sigma}_B}\right)^m \left(\frac{\sigma-\sigma_G}{\sigma-\tilde{\sigma}_G}\right)^n ,$$

The flow is illustrated in Figures 14a and 14b.

d) Flow about an angle plate. This is obtained with

$$\frac{dz}{dw} = C \left(\frac{\sigma - \bar{\sigma}_A}{\sigma - \sigma_A} \right)^m$$

where $m\pi$ is the plate angle in radians. The flow is shown in Figures 15a and 15b.

e) Flow about an aerofoil with flat and constant pressure boundary. This is achieved with

$$\frac{dz}{dw} = C \left(\frac{\sigma - \bar{\sigma}_A}{\sigma - \sigma_A} \right) \left(\frac{\sigma - \bar{\sigma}_G}{\sigma - \sigma_G} \right)^m$$

where $-m\pi$ is the trailing edge angle of the aerofoil. The flow as plotted on a computer is given in Figure 16a and 16b. This result was obtained previously by Hurley and Ruglen [3] (using Helmholtz free stream-line theory but not with the σ plane technique) and subsequently recalculated by Saffman and Tanveer [4]. Comparing this flow with the separated flow with reunion about a flat plate it can be seen that there are three equations connecting β and γ. This redundant system of equations imposes certain relationship between σ_A, σ_G, and σ_D and consequently between z_C, z_G, and z_E, as previously noted in Refs 3 and 4.

f) By setting m equal to unity in the last case the flow about a flat plate with leading edge bubble results. This is illustrated in Figures 17a and 17b.

g) By placing one more singularity at the point B on the semi-circle, making

$$\frac{dz}{dw} = \left(\frac{\sigma - \bar{\sigma}_A}{\sigma - \sigma_A} \right) \left(\frac{\sigma - \sigma_B}{\sigma - \bar{\sigma}_B} \right) \quad ,$$

The flow around a flat plate with a different type of separation bubble and no separation from the trailing edge results, as shown in Figures 18a and 18b.

It should be noted that in these variants the closing condition (7) is still required whenever σ_D is inside the semi-circular disc.

5.   CONCLUSIONS

The incompressible, inviscid flow about a flat plate and certain of its variants has been described using a single σ-plane. Some of the interesting points from the flows about a flat plate are:

-     For every angle of attack the flow can vary between full Joukowski flow to full Helmholtz flow. Reduced circulation flow, in which the free streamlines close at a finite point, and Helmholtz - Schmieden's flows are the intermediate states.

-     Lift and drag vary continuously from one possible state to another possible state, the drag being zero for Joukowski and reduced circulation flows.

-     There is only one inflection point on the free stream-lines. The inflection point is on the free stream-line leaving the leading edge if the flow is open and is on the free stream-line, leaving the trailing edge if the flow closes.

A proof is given showing that analogous results also hold for an arbitrary aerofoil.

Using the σ-plane technique the flows about a range of variants of the flat plate are presented.

From the applications presented it would seem that the σ-plane technique would also be appropriate for the solution of two-dimensional free streamline flows about an arbitrary single body.

REFERENCES

1. SCHMIEDEN, C., "Flow around wings accompanied by separation of vortices", NACA-TM. 961 translation, Luftfahrtforschung 17, No. 2 (1940).

2. VALLENTINE, H.R., "Applied hydrodynamics", Butterworth (1969).

3. HURLEY, D.G., and RUGLEN, N., "The use of boundary layer control to establish free stream-line flows being an introduction to the free stream-line flap," ARL-Report A109, Australia (1958).

4. SAFFMAN, P.G., and TANVEER, S.,"Vortex Induced Lift on Two dimensional low speed wings",S.I.A.M. (1984), 65-78.

5. SOUTHWELL, R.V., and VAISEY, G., "Relaxation Method applied to Engineering Problems", Phil. Trans. R. Soc. Lond. A 240 (1946), 117-160.

z - plane
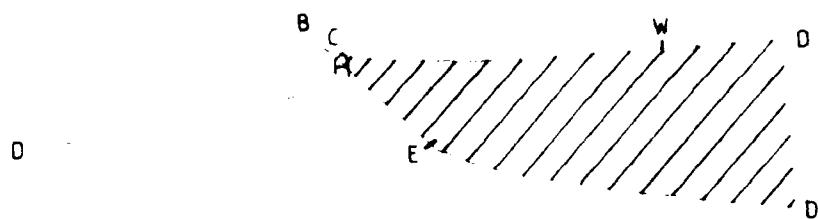
FIG. 1.   SEPARATED FLOW ABOUT A FLAT PLATE
IN THE Z-PLANE.   (ANGLE OF ATTACK = $\pi/4$)
COMPUTED RESULT
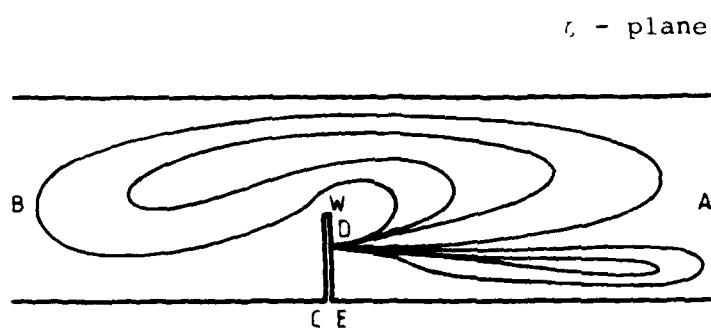
ζ - plane

FIG. 2.   SEPARATED FLOW ABOUT A FLAT PLATE
REPRESENTED IN THE ζ PLANE
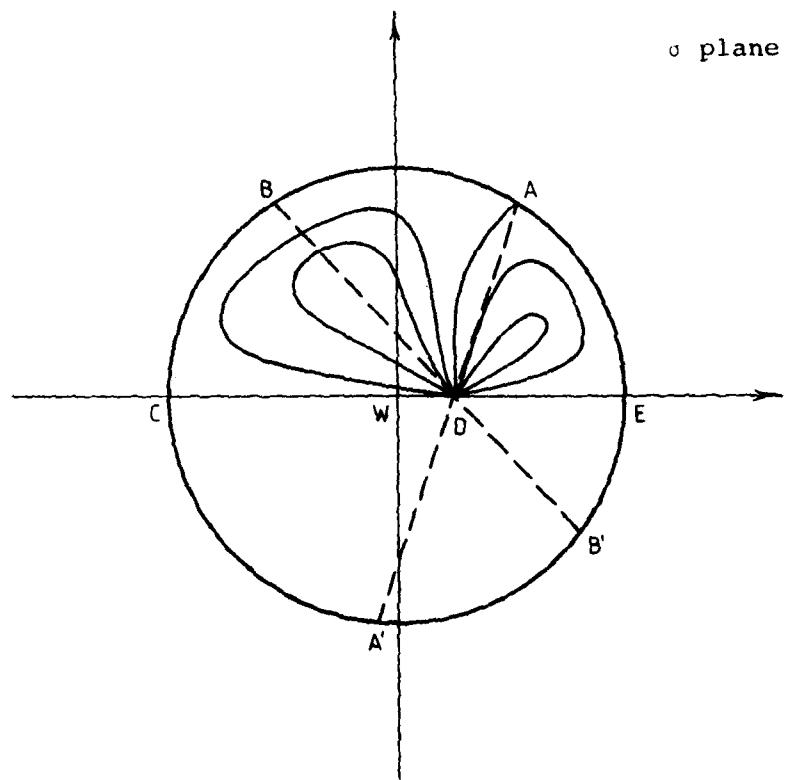
FIG. 3.   SEPARATED FLOW ABOUT A FLAT PLATE
REPRESENTED IN THE σ PLANE
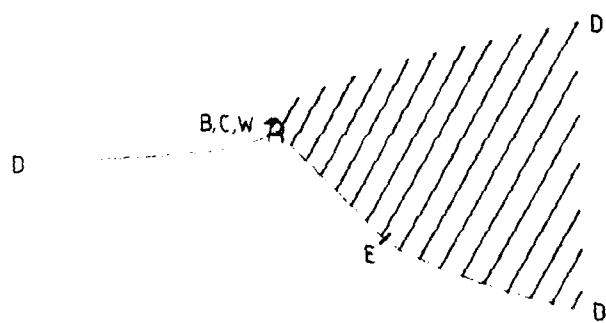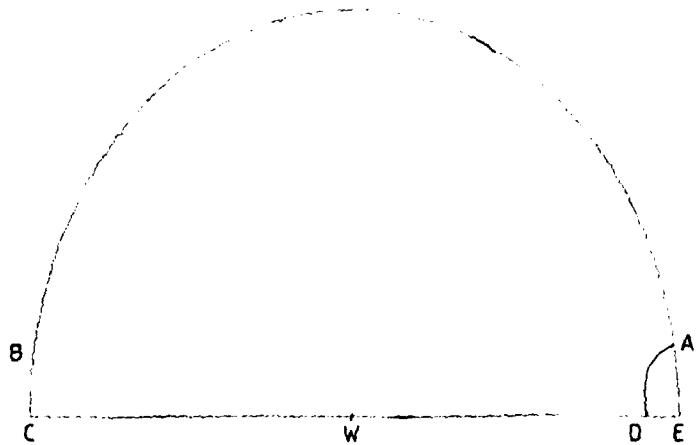
z - plane

σ plane

FIG. 4.   SEPARATED FLOW ABOUT A FLAT PLATE,
          WITH $\sigma_D \approx \sigma_E$.   (ANGLE OF ATTACK = $\pi/4$)
          COMPUTED RESULT

z - plane
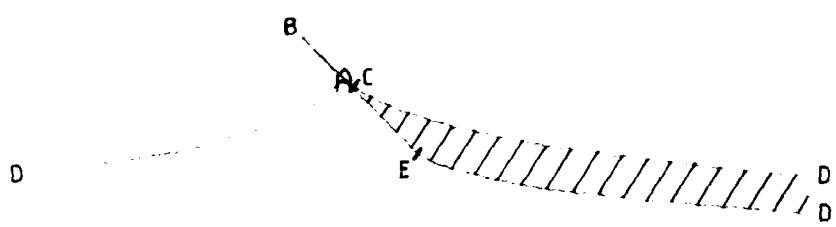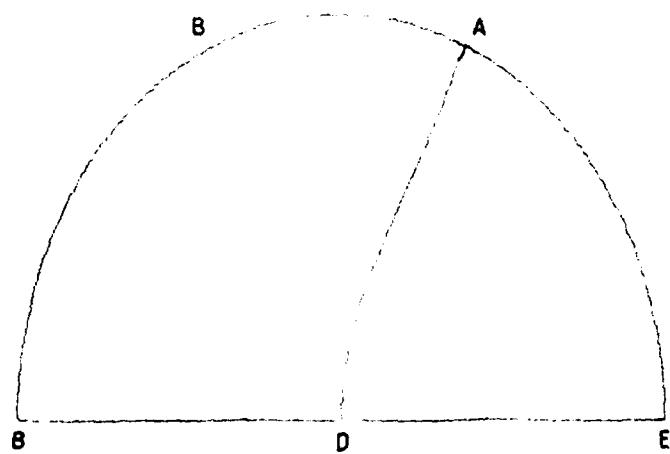
σ plane

FIG. 5.   SEPARATED FLOW ABOUT A FLAT PLATE,
WITH $\sigma_D \approx \sigma_W$.   COMPUTED RESULT
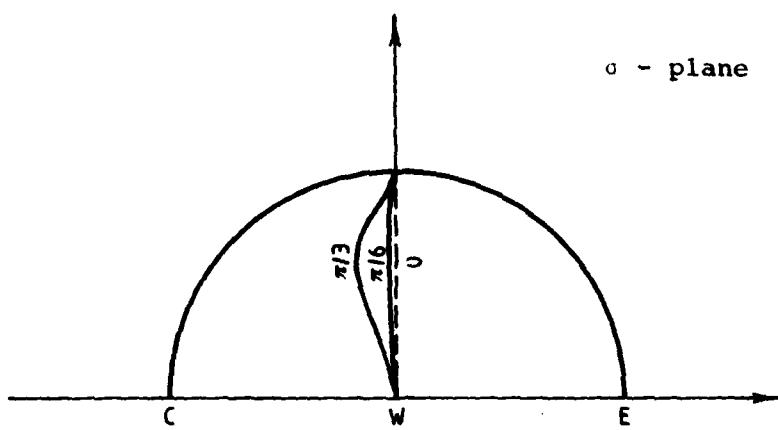
σ - plane

FIG. 6.   COMPUTED LOCUS OF σ_n FOR ANGLES OF
            ATTACK $\alpha = 0$, $\pi/6$, $\pi/3$
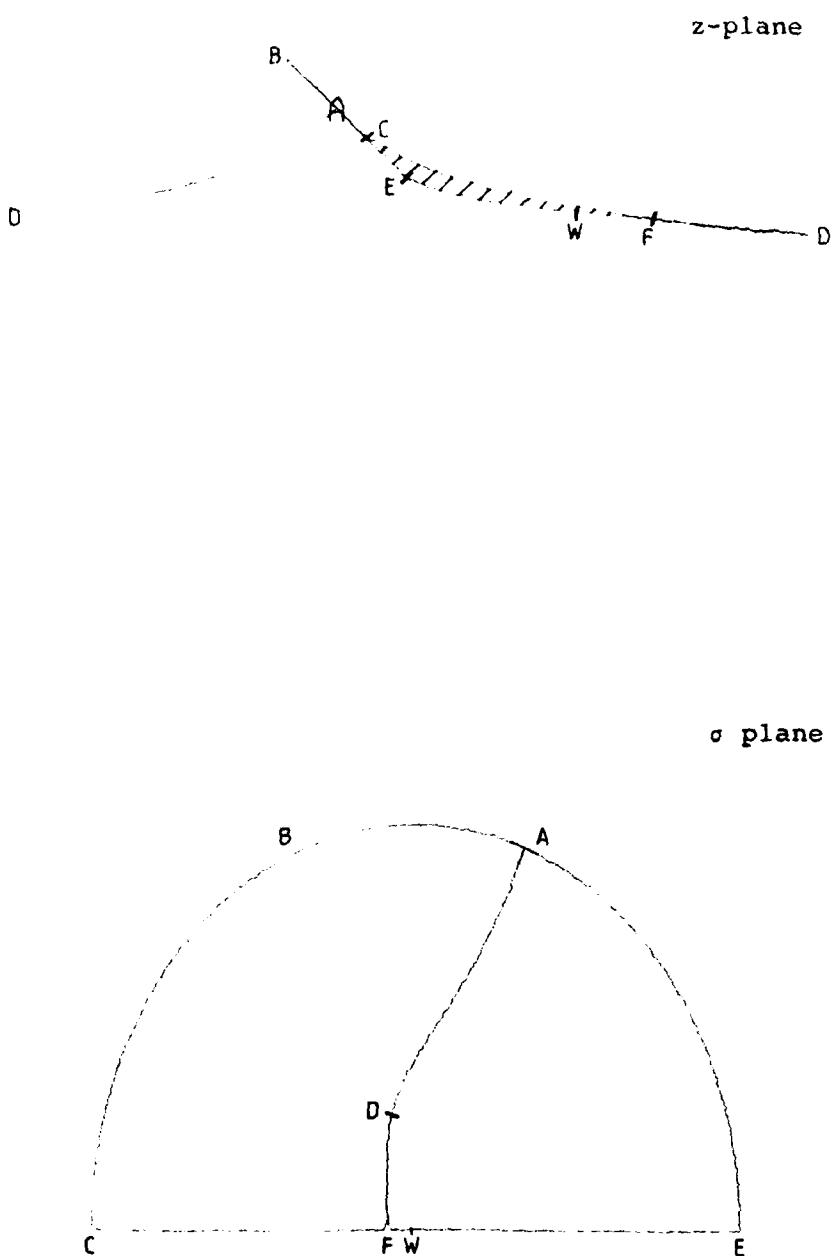
z-plane

σ plane

FIG. 7.   COMPUTED FLOW ABOUT AN INCLINED FLAT PLATE
WITH REAR SEPARATION BUBBLE   (ANGLE OF
ATTACK = π / 4)

z - plane

B

A

C

E   F

D

D

σ plane

B   A

D
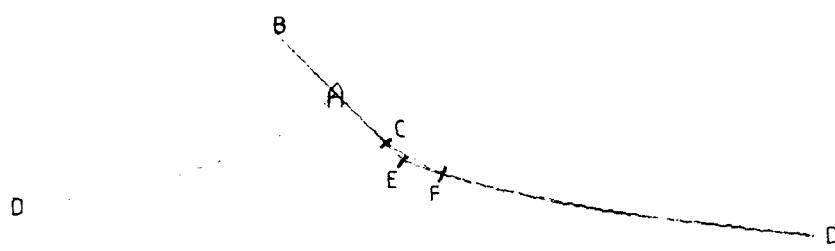
C   F W   E
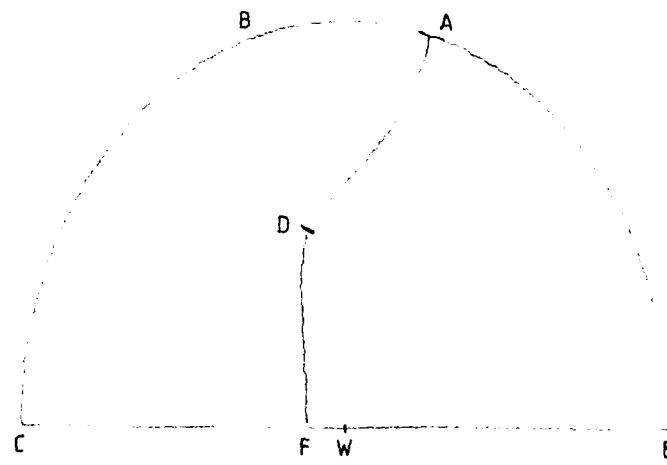
FIG. 8   FLOW ABOUT AN INCLINED FLAT PLATE
            (ANGLE OF ATTACK = π/4) WITH VERY
            SHORT REAR SEPARATION BUBBLE.
            COMPUTED RESULT

z - plane
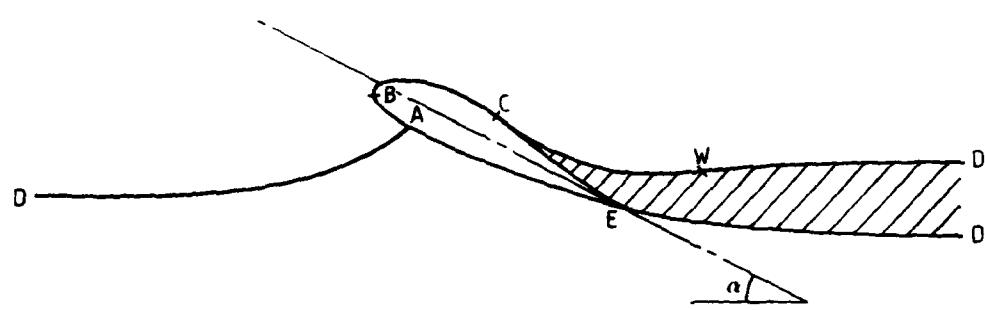
FIG. 9   SEPARATED FLOW ABOUT AN ARBITRARY
            AEROFOIL.
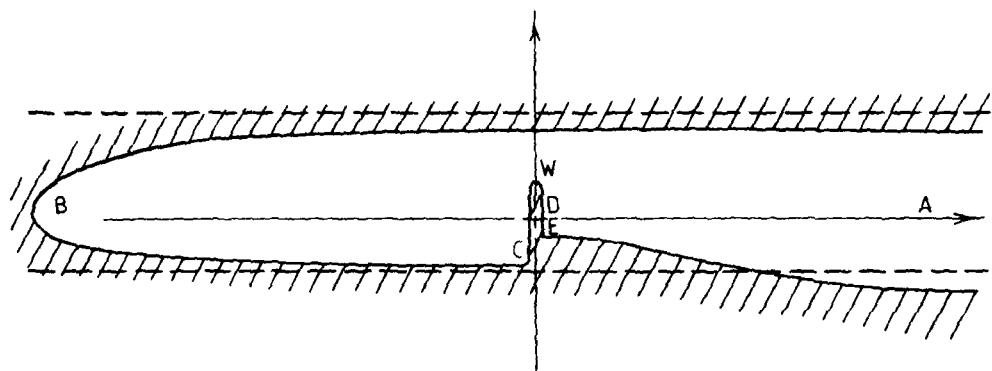
FIG. 10.   SEPARATED FLOW ABOUT AN ARBITRARY AEROFOIL,
REPRESENTED IN THE ζ - PLANE
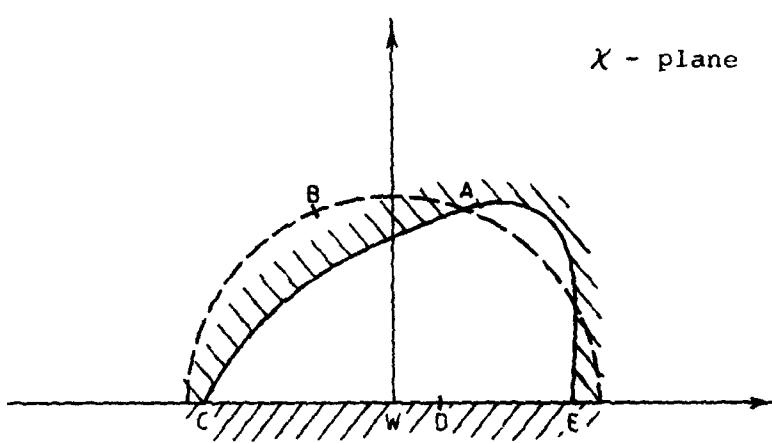
χ - plane

FIG. 11.   SEPARATED FLOW ABOUT AN ARBITRARY
AEROFOIL, REPRESENTED IN THE χ - PLANE
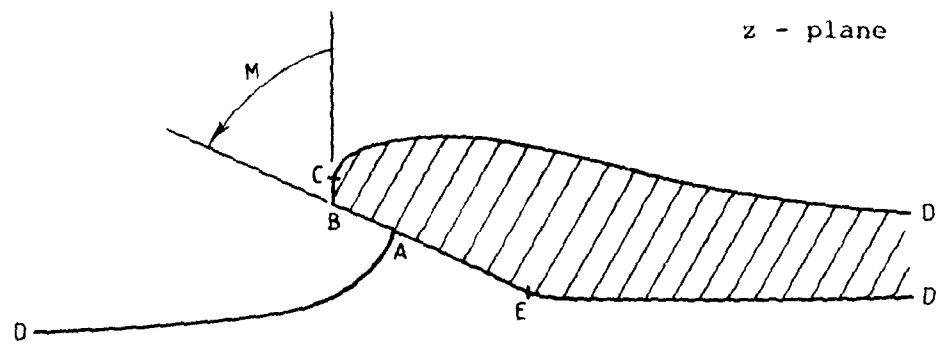
z - plane

FIG. 12(a)   SKETCH OF SEPARATED FLOW ABOUT
   A FLAT PLATE WITH FRONT FLAP
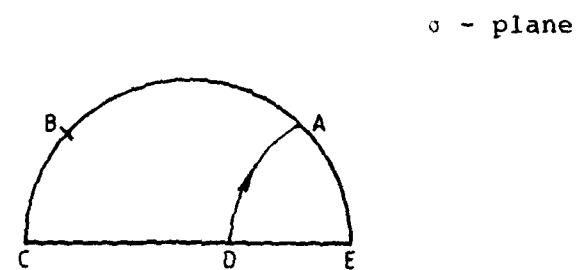
σ - plane

FIG. 12(b)   SKETCH OF THE σ - PLANE REPRESENTATION
   OF SEPARATED FLOW ABOUT A FLAT PLATE
             WITH FRONT FLAP

z - plane
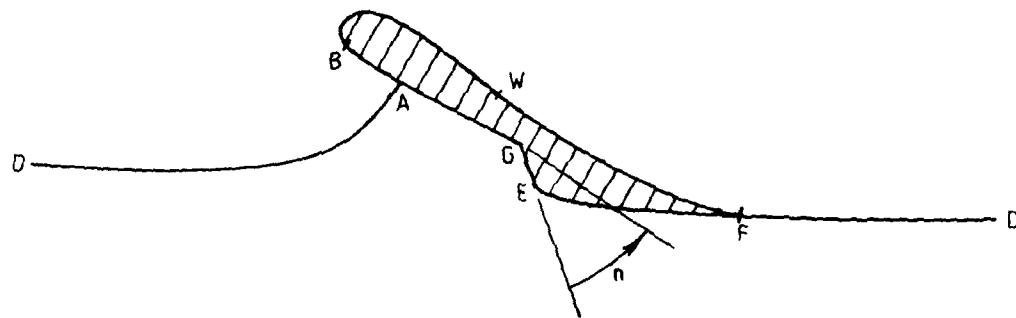
FIG. 13(a)   SKETCH OF SEPARATED FLOW ABOUT A
FLAT PLATE WITH REAR FLAP
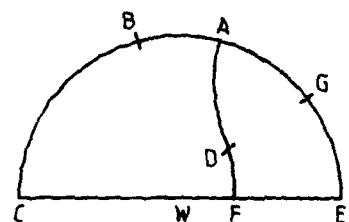
o - plane

FIG. 13(b)   SKETCH OF THE ɢ - PLANE REPRESENTATION
OF SEPARATED FLOW ABOUT A FLAT FLAP
WITH REAR FLAP

z - plane
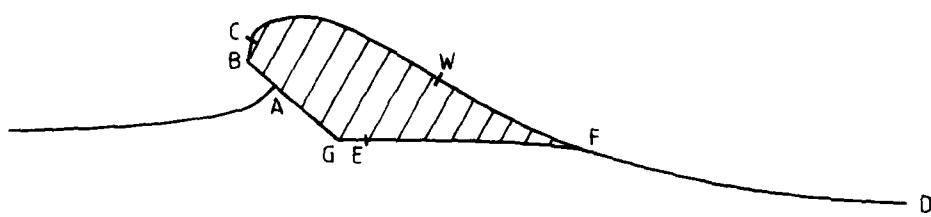
FIG. 14(a)   SKETCH OF SEPARATED FLOW ABOUT
A FLAT PLATE WITH FRONT AND REAR
FLAPS
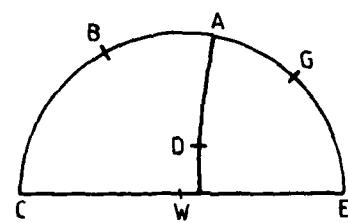
σ - plane

FIG   14(b)   SKETCH OF THE σ - PLANE REPRESENTATION
OF SEPARATED FLOW ABOUT A FLAT PLATE
WITH FRONT AND REAR FLAPS

z – plane
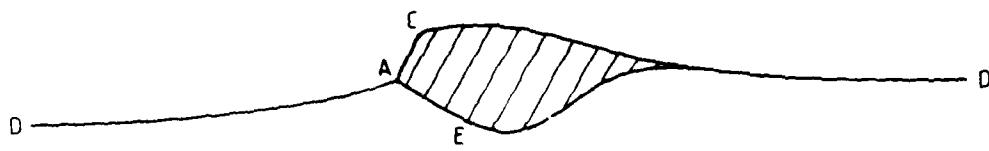
FIG. 15(a)   SKETCH OF SEPARATED FLOW
ABOUT AN ANGLE PLATE
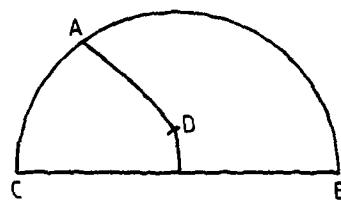
σ – plane

FIG. 15(b)   SKETCH OF THE σ – PLANE REPRESENTATION
OF SEPARATED FLOW ABOUT AN ANGLE PLATE

z - plane
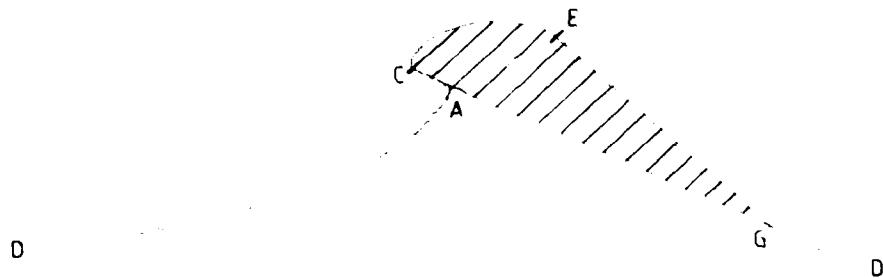

FIG. 16(a)   COMPUTER PLOT OF A FLAT PLATE
             WITH REAR SEPARATION-FLAP
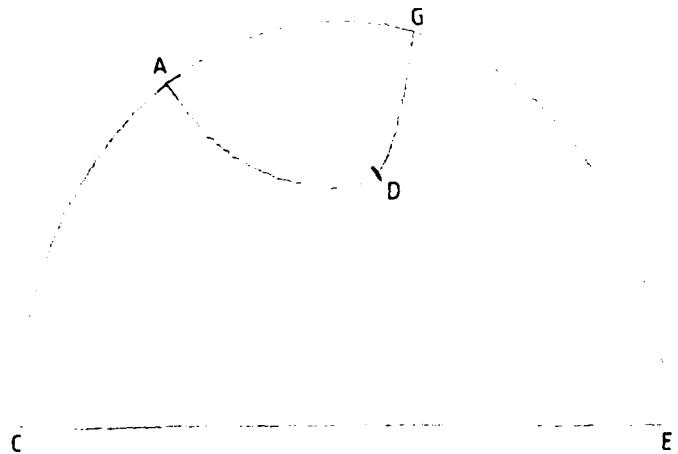
σ plane


FIG. 16(b)   COMPUTER PLOT OF THE σ - PLANE
             REPRESENTATION OF FLOW ABOUT A
             FLAT PLATE WITH REAR
             SEPARATION - FLAP
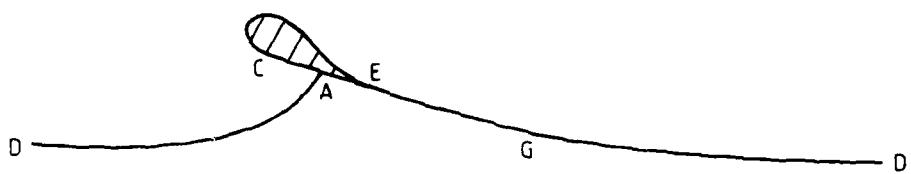
FIG. 17(a)   SKETCH OF FLOW ABOUT A FLAT
             PLATE WITH LEADING EDGE BUBBLE
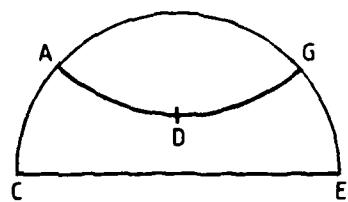
σ - plane

FIG. 17(b)   SKETCH OF THE σ - PLANE REPRESENTATION
             OF FLOW ABOUT A FLAT PLATE WITH
   LEADING EDGE BUBBLE
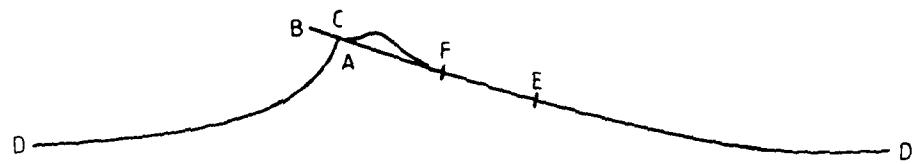
FIG. 18(a) SKETCH OF FLOW ABOUT A FLAT
PLATE WITH MIDDLE SEPARATION
BUBBLE
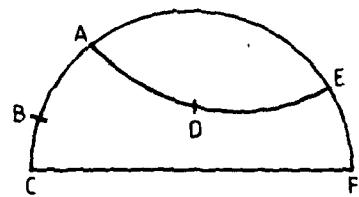
ʋ - plane

FIG. 18(b) SKETCH OF THE ʋ - PLANE REPRESENTATION
OF FLOW ABOUT A FLAT PLATE WITH
MIDDLE SEPARATION BUBBLE

AUSTRALIA

Department of Defence

Defence Central

Chief Defence Scientist   )
Deputy Chief Defence Scientist   )
Superintendent, Science and Program Administration) (1 copy)
Controller, External Relations, Projects and   )
    Analytical Studies   )
Counsellor, Defence Science (USA)   (Doc Data sheet only)
Defence Central Library
Document Exchange Centre, DISB   (18 copies)
Joint Intelligence Organisation
Librarian H Block, Victoria Barracks, Melbourne
Director General - Army Development (NSO)   (4 copies)
Defence Science Adviser (UK) (Doc Data sheet only)

Aeronautical Research Laboratories

Director
Library
Superintendent - Aerodynamics
Divisional File - Aerodynamics
Authors: T. Tran-Cong
          C. Martin
D.A. Secomb
M. Cooper
J.S. Drobik
R.A. Feik
N. Pollock
B. Fairlie
P. Gibbens

Materials Research Laboratories

Director/Library

Defence Research Centre

Library

RAN Research Laboratory

Library

Navy Office

    Navy Scientific Adviser
    RAN Aircraft Maintenance and Flight Trials Unit
    Directorate of Naval AircraftEngineering
    Directorate of Naval Aviation Policy
    Superintendent, Aircraft Maintenance and Repair

Army Office

    Scientific Adviser - Army
    Engineering Development Establishment, Library
    Royal Military College Library

Air Force Office

    Air Force Scientific Adviser
    Aircraft Research and Development Unit
        Scientific Flight Group
        Library
    Technical Division Library
    Director General Aircraft Engineering - Air Force
    Director General Operational Requirements - Air Force
    HQ Operational Command (SMAINTSO)
    HQ Support Command (SLENGO)
    RAAF Academy, Point Cook

Central Studies Establishment

    Information Centre

Government Aircraft Factories

    Manager
    Library

Department of Science and Technology

    Bureau of Meteorology, Library

Department of Aviation

    Library
    Flight Standards Division

Universities and Colleges

Sydney	Dr G.P. Steven, Department of Aeronautical Engineering
Wollongong	Prof. J.R. Blake, Head, Department of Mathematics
NSW	Prof. V.T. Buckwald, Dean, Faculty of Science

FRANCE

ONERA, Library

GERMANY

DFVLR Braunschweig, Library

UNITED KINGDOM

Royal Aircraft Establishment, Bedford, Library
Royal Aircraft Establishment, Farnborough, Library

Universities and Colleges

Cranfield Inst. of Technology, Library

SPARES   (20 copies)

TOTAL   (94 copies)

DOCUMENT CONTROL DATA

1 a. AR No	1 b Establishment No	2. Document Date	3. Task No
AR-004-021	ARL-AERO-TM-372	MAY, 1985	DST 84/037

4. Title	5. Security a. document	6. No Pages
TWO-DIMENSIONAL HELMHOLTZ FLOW ABOUT A FLAT PLATE AND RELATED SHAPES	UNCLASSIFIED	13
	b. title c. abstract	7. No Refs
	U U	5

8. Author(s)	9. Downgrading Instructions
T. TRAN-CONG, C.A. MARTIN	-

10. Corporate Author and Address	11. Authority (as appropriate) a.Sponsor b.Security c.Downgrading d.Approval
Aeronautical Research Laboratories P.O. Box 4331, Melbourne, Vic. 3001	

12. Secondary Distribution (of this document)

Approved for Public Release

Overseas enquirers outside stated limitations should be referred through ASDIS, Defence Information Services Branch, Department of Defence, Campbell Park, CANBERRA ACT 2601

13. a. This document may be ANNOUNCED in catalogues and awareness services available to ...

No limitations

13. b. Citation for other purposes (ie casual announcement) may be (select) unrestricted(or) as for 13 a.

14. Descriptors	15. COSATI Group
Freestream Mathematical model Flat plates Airfoil	01000 12010

16. Abstract

Various configurations of two-dimensional Helmholtz flow about a flat plate are obtained with a free streamline theory using the flow inside a semi-circular disc;   The flow can vary continuously from fully-attached Joukowski flow to the commonly-known fully-separated Helmholtz flow. Some general results for an arbitrary aerofoil are also obtained. Variations of the technique give the flow about flat plates with flaps and other related configurations.

PF 65

This page is to be used to record information which is required by the Establishment for its c   use but which will not be added to the DISTIS data base unless specifically requested.

16. Abstract (Contd)

17. Imprint
Aeronautical Research Laboratories, Melbourne

18. Document Series and Number	19. Cost Code	20. Type of Report and Period Covered
Aerodynamics Technical Memorandum 372	52 7740	

21. Computer Programs Used

22. Establishment File Ref(s)

END

DATE
FILMED

12-85

DTIC